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Solution by HENRY HEATON, Belfield, N. D.

By making the line the circumference of a circle it will be seen that each part has exactly the same chance, so that the average of one part is the same as that of any other. Hence it will be sufficient to find the average length of any one part.

Let x_1, x_2, ..., x_{n-1} be the distance of the different points from one extremity of the line. Then the average length of $(l-x_{n-1})^p$ is

$$A=\frac{\int_0^l\int_0^{x_{n-1}}\dots\int_0^{x_2}(l-x_{n-1})^p dx_{n-1}dx_{n-2}\dots dx_1}{\int_0^l\int_0^{x_{n-1}}\dots\int_0^{x_2}dx_{n-1}dx_{n-2}\dots dx_1}$$

$$=\frac{\int_0^l(l-x_{n-1})^p x_{n-1}^{n-2}\,dx_{n-1}}{\int_0^l x_{n-1}^{n-2}dx_{n-1}}=(n-1)l^p B[(p+1),\ (n-1)]$$

$$=\frac{(n-1)l^p\,[\Gamma(p+1),\ \Gamma(n-1)]}{\Gamma(p+n)}=\frac{l^p\,[\Gamma(p+1),\ \Gamma(n)]}{\Gamma(p+n)}$$

or, the average length of $x_1{}^p$ is

$$A=\frac{\int_0^l\int_0^{x_{n-1}}\dots\int_0^{x_2}dx_{n-1}\,dx_{n-2}\dots x_1{}^p dx_1}{\int_0^l\int_0^{x_{n-1}}\dots\int_0^{x_2}dx_{n-1}\,dx_{n-2}\dots dx_1}$$

$$=\frac{l^p(n-1)!\,p!}{(p+n-1)!}=\frac{l^p[\Gamma(n),\ \Gamma(p+1)]}{\Gamma(p+n)}.$$

Also solved by G. B. M. Zerr.

186. Proposed by G. B. M. ZERR, Ph. D., Parsons, W. Va.

An urn contains $n=100$ balls; $a=25$ balls are stamped, at random, with the letter A; $b=30$ balls are stamped, at random, with the letter B; $c=40$ balls are stamped, at random, with the letter C; $d=50$ balls are stamped, at random, with the letter D. One ball is drawn at random; find the chance it has on it no letter, the letter A, or B, or C, or D, or the letters AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, or $ABCD$.

Solution by J. E. SANDERS, Reinersville, O., and the PROPOSER.

$(a/n)=\frac{25}{100}=\frac{1}{4}$, $(1-a/n)=\frac{3}{4}$; $(b/n)=\frac{30}{100}=\frac{3}{10}$, $(1-b/n)=\frac{7}{10}$; $(c/n)=\frac{40}{100}=\frac{2}{5}$, $(1-c/n)=\frac{3}{5}$, $(d/n)=(1-d/n)=\frac{50}{100}=\frac{1}{2}$.

$\therefore$ The required chances are in order:

$(1-a/n)(1-b/n)(1-c/n)(1-d/n)=\frac{3}{4}\cdot\frac{7}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{63}{400}$.
$(a/n)(1-b/n)(1-c/n)(1-d/n)=\frac{1}{4}\cdot\frac{7}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{21}{400}$.
$(1-a/n)(b/n)(1-c/n)(1-d/n)=\frac{3}{4}\cdot\frac{3}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{27}{400}$.
$(1-a/n)(1-b/n)(c/n)(1-d/n)=\frac{3}{4}\cdot\frac{7}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{42}{400}$.
$(1-a/n)(1-b/n)(1-c/n)(d/n)=\frac{3}{4}\cdot\frac{7}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{63}{400}$.
$(a/n)(b/n)(1-c/n)(1-d/n)=\frac{1}{4}\cdot\frac{3}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{4}{400}$.
$(a/n)(1-b/n)(c/n)(1-d/n)=\frac{1}{4}\cdot\frac{7}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{14}{400}$.
$(a/n)(1-b/n)(1-c/n)(d/n)=\frac{1}{4}\cdot\frac{7}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{21}{400}$.
$(1-a/n)(b/n)(c/n)(1-d/n)=\frac{3}{4}\cdot\frac{3}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{18}{400}$.
$(1-a/n)(b/n)(1-c/n)(d/n)=\frac{3}{4}\cdot\frac{3}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{27}{400}$.
$(1-a/n)(1-b/n)(c/n)(d/n)=\frac{3}{4}\cdot\frac{7}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{42}{400}$.
$(a/n)(b/n)(c/n)(1-d/n)=\frac{1}{4}\cdot\frac{3}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{6}{400}$.
$(a/n)(b/n)(1-c/n)(d/n)=\frac{1}{4}\cdot\frac{3}{10}\cdot\frac{3}{5}\cdot\frac{1}{2}=\frac{9}{400}$.
$(a/n)(1-b/n)(c/n)(d/n)=\frac{1}{4}\cdot\frac{7}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{14}{400}$.
$(1-a/n)(b/n)(c/n)(d/n)=\frac{3}{4}\cdot\frac{3}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{18}{400}$.
$(a/n)(b/n)(c/n)(d/n)=\frac{1}{4}\cdot\frac{3}{10}\cdot\frac{2}{5}\cdot\frac{1}{2}=\frac{6}{400}$.

187. Proposed by HENRY HEATON, Belfield, N. D.

Through every point of a given square straight lines are drawn in every possible direction, terminating in the sides of the square. What is the average length of such lines?

Solution by the PROPOSER.

Let x and y be the coordinates of one of the points, and θ=the angle one of the lines makes with the side taken as the Y-axis. Then the length of the line from the point to the side taken as the X-axis is $y\sec\theta$.

It is only necessary to find the average length of this line, for the average length of lines drawn to the other sides is evidently the same.

When the line passes through the origin, $y\tan\theta=x$. Hence the limits of θ are 0 and $\tan^{-1}(x/y)$. Hence, the required average is,

$$A=\int_0^a\int_0^a\int_0^{\tan^{-1}(x/y)} y\sec\theta\,dx\,dy\,d\theta\div\int_0^a\int_0^a\int_0^{\tan^{-1}(x/y)} dx\,dy\,d\theta$$

$$=\int_0^a\int_0^a \log\left(\frac{x}{y}+\frac{\sqrt{(x^2+y^2)}}{y}\right)dxy\,dy\div\int_0^a\int_0^a \tan^{-1}\left(\frac{x}{y}\right)dx\,dy$$

$$=\int_0^a\left[\frac{a^2}{2}\log\left(\frac{x+\sqrt{(a^2+x^2)}}{a}\right)+\frac{x}{2}\sqrt{(a^2+x^2)}-\frac{x^2}{2}\right]dx$$

$$\div\int_0^a\left[a\tan^{-1}\left(\frac{x}{a}\right)+\frac{x}{2}\log\left(\frac{a^2+x^2}{x^2}\right)\right]dx=\frac{2a}{\pi}[\log(1+\sqrt{2})-\tfrac{1}{3}(\sqrt{2}-1)]$$

$$=.4095a.$$

Also solved by G. B. M. Zerr, whose solution will appear in the next issue.